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VARIOUS ASPECTS OF NUMERICAL DETERMINATION OF NUTATION CONSTANTS. I. IMPROVEMENT OF RIGID-EARTH NUTATION

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ABSTRACT

Kinoshita's (1977) nutation for the rigid Earth has been improved in the following ways: (1) H, K_m, and K_s values of Seidelmann (1982) are used; (2) second-order effects of the coupling between rotation of the Earth and orbital motion of the Moon have been added, besides the 1.21 mas in $\Delta\psi_2$ and -0.24 mas in $\Delta\epsilon_2$; two near-semiannual terms with periods equal to 187.7 and 177.8 days are found; their amplitudes amount to a 0.1 mas level; (3) the calculation is carried to 0.01 mas. In addition to the original 106 periodic terms, another 156 terms are added. In 18 terms the differences between our result and that of Kinoshita (1977) or Kinoshita and Souchay (1988) amount to more than 0.05 mas. As a by-product, the relation between tidal potential and nutation theory originally emphasized by Melchior has been studied. Oppolzer terms are shown to be the contribution of the Chandler Wobble normal mode, and their numerical values are derived from given tidal-potential amplitudes. Actual computation shows that nutation values derived from the tidal potential are equivalent to those obtained from the astronomical procedure to 0.01 mas for most periodic terms.

I. INTRODUCTION

Nutation, together with precession, represents the motion of the Earth's axis with respect to an inertial system. Up to 1984, the numerical values of nutation series were based on Woolard's theory (Woolard 1953), and the relevant Earth axis was the rotation axis. From optical observations obtained over many decades, an error of a few mas was found in Woolard's nutation (see Kinoshita *et al.* 1979). The IAU (1980) theory of nutation has been adopted as a successor to Woolard's, and the corresponding Earth axis is the Celestial Ephemeris Pole. Nutation of a rigid Earth has been based on Kinoshita's study (Kinoshita 1977), which was then modified for a deformable Earth in Wahr's theory (Wahr 1981a). The whole evaluation is clearly summarized in Seidelmann (1982). Numerically, the differences between the IAU (1980) nutation and that of Woolard amount to 50 mas, and are larger than 5 mas in eight terms. The differences between Kinoshita's and Woolard's values are comparatively small: only in six terms are corrections larger than 0.5 mas, with the largest at 1.7 mas. Clearly, the most significant variations are caused by deformation of the Earth, especially the ellipticity of the liquid core.

When compared with classical optical observations, new techniques like VLBI have about two orders of magnitude improvement in accuracy. In a relatively short time, VLBI observations hve shown that the newly adoped IAU (1980) nutation may have errors at a level of a few mas (Herring *et al.* 1986). Investigations searching for geophysical explanations have been stimulated (see Gwinn *et al.* 1986; Dehant 1987a, b; Wahr and Bergen 1986; Wahr and de Vries 1989; Mathews *et al.* 1988; de Vries and Wahr 1988; Yoder and Ivins 1987; etc). Up to now, the nutation series has been derived from theory alone. Only the observed value of the principal nutation in obliquity (or of lunisolar precession) has been used to determine a scalelike factor. Our knowledge of the planetary motion (of Sun and Moon) and of the Earth's interior was good enough compared with the accuracy of observations at that time. Now, with VLBI observations some major periods of nutation series can be determined better empirically rather than by theoretical derivation. Observed nutation corrections can be used to deepen our understanding of the Earth's interior. This initiates a new chapter in the study of nutation.

The VLBI-determined nutation correction can be written in the form

$$\delta\psi_v = \Delta\psi_{v\mathrm{VLBI}} - \Delta\psi_{v\mathrm{IAU(1980)}},$$

and similarly for $\delta\epsilon_v$; here v is the consecutive term number in the IAU (1980) nutation series. At present, terms with $v = 1, 9, 10, 31$ have been estimated from observations. Except for $v = 1$, the quoted nominal standard errors reach 0.02 mas for $\delta\epsilon_v$ and 0.07 mas for $\delta\psi_v$ (see BIH Annual report for 1987, D-111). In fact, $\delta\psi_v$ and $\delta\epsilon_v$ are the combined contribution of: (1) observation error; (2) imperfections of the rigid-Earth model; (3) Wahr's computation; and (4) other geophysical reasons. Careful investigation of the first three causes may be helpful to the study of the fourth. The former is the main aim of our research.

In this paper, only the nutation of the rigid Earth is treated. The goal is to improve Kinoshita's (1977) numerical results. Since many basic conceptions and equations are quoted from his paper, for brevity, the notation K77 is used in referring to this paper. Much attention is paid to numerical, rather than theoretical, aspects.

As pointed out by Seidelmann (1987): "There is a need to increase the accuracy of the nutation theory. This involves a completely new determination of the fundamental nutation theory to more significant figures and will involve inclusion of many more terms in the theory." Our results are rounded to 0.01 mas, and as a result about a hundred more terms are included.

II. BASIC EQUATIONS OF KINOSHITA'S THEORY

Nutations (of the angular-momentum axis) for a rigid Earth have the following forms K77:

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$$\Delta\psi = K \sum_{v\neq 0} \left(\frac{E_v}{N_v}\right) \sin \vartheta_v , \qquad (1)$$

$$\Delta\epsilon = -\left(\frac{K}{\sin \epsilon}\right) \sum_{v\neq 0} (i_5)\left(\frac{B_v}{N_v}\right)\cos \vartheta_v ,$$

where the common factor K is expressed as

$$K_m = 3H\,[\mathscr{M}_m/(\mathscr{M}_m + \mathscr{M}_E)]\,n_m^2/(F_2^3\omega),$$
$$K_s = 3H\mathscr{M}_s/(\mathscr{M}_s + \mathscr{M}_E + \mathscr{M}_m)(n_s^2/\omega), \qquad (2)$$

and

$$B_v = -(3\cos^2\epsilon - 1)A_v^0/6 + \sin 2\epsilon A_v^1/2 - \sin^2\epsilon A_v^2/4,$$
$$E_v = (A_v^0 - A_v^2/2)\cos\epsilon + (\cos 2\epsilon/\sin\epsilon)A_v^1, \qquad (3)$$
$$v = (i_1, i_2, i_3, i_4, i_5),$$
$$\vartheta_v = i_1 1 + i_2 1' + i_3 F + i_4 D + i_5 \Omega.$$

A_v^i are quantities depending on the orbital elements of the Moon and the Sun. Explanations for all other notations are omitted for brevity. To obtain the nutations of the figure axis, the so-called Oppolzer terms must be added. Note that whenever we use a semimajor axis of the orbit of the Moon equal to a_{Brown}/F_2 for developing A_i, the F_2^3 in Eq. (2) for computing K_m should be removed. B_v and E_v are functions of ϵ, where ϵ is the mean obliquity of date. The rates of nutation $\Delta\dot\psi$ and $\Delta\dot\epsilon$ are basically caused by the rate of ϵ, the currently adopted value of which is $\dot\epsilon_{J2000} = d\epsilon/dt = -46''.8150$/century (Lieske *et al.* 1977). In addition, the rates of A_v^i give some contribution to $\Delta\dot\psi$ and $\Delta\dot\epsilon$: They especially affect three terms caused by the action of the Sun.

III. REPETITION OF KINOSHITA'S COMPUTATION

In K77, A_v^i are calculated from Brown's theory of the Moon as improved by Eckert *et al.* (1966) and Newcomb's theory of the Sun. In Kinoshita and Souchay (1988), they are evaluated by using VSOP82 and ELP2000 (Chapront and Chapront 1983). First, we simply repeat Kinoshita's calculation by adopting his K and A_v^i values to estimate $\Delta\psi$, $\Delta\epsilon$ for the angular-momentum axis and the Oppolzer terms. The adopted epoch is J_{2000} with $\epsilon_{2000} = 23°26'21''.448$. A few printing errors in K77 have been removed (e.g., A_{41}^1 should be $-84\,156$—the original value was -4156, and A_{91}^1 has an opposite sign). The computation of $\Delta\psi_1$ deserves some explanation. According to Hori's theory (1966),

$$\vartheta_v = i_1 1 + i_2 1' + i_3 F + i_4 D + i_5(\Omega_0 - h^*),$$

while N_v is defined as

$$N_v = i_1 n_m + i_2 n_s + i_3 \frac{dF}{dt} + i_4 \frac{dD}{dt} + i_5 n_\Omega$$

without the precession dh^*/dt (see also Aoki 1988). Therefore the term $\partial(1/N_v)/\partial I$ of Eq. (6.17) in K77, which amounts to 2.95 mas, is a second-order effect [cf. Eq. (D.6a) of Aoki 1988]. K77 has adopted Kubo's version of perturbation theory to calculate $\Delta\psi_1$ instead of Hori's method (Kinoshita 1989). Accordingly, this term should be excluded in the later consideration of the second-order effects. After this has been taken into account, our results are equivalent to those of K77 at the level of 0.1 mas for all 106 terms.

Now, we compare our results with those of Kinoshita and Souchay (1988). Except for $\Delta\psi_1$ and $\Delta\epsilon_1$, the differences are at the level of 0.02 mas or less. There are seven terms, whose differences lie between 0.01 and 0.02 mas, while the rest are less than 0.01 mas. From this, one sees that although ELP2000 and VSOP82 may be much better than Brown's table (improved by Eckert *et al.*) and Newcomb's theory, their effects on nutation are basically limited to $\Delta\psi_1$ and $\Delta\epsilon_1$.

IV. PROBLEM OF THE VALUE OF K

In K77 and Kinoshita and Souchay (1988), the K values are different:

$K_m = 7567''.72161$, $K_s = 3475''.39770$ (in 1977),

$K_m = 7547''.1635$*, $K_s = 3475''.3975$ (in 1988),

The asterisk indicates a value obtained by using $a = a_{\text{Brown}}/F_2$ for deriving A_i, which should be divided by F_2^3 in order to compare it with K_m of 1977. The corresponding value is 7567.7334. The K numbers given in the IAU report (Seidelmann 1982) are

$K_m = 7567''.8292$, $K_s = 3475''.4416$.

The relative difference between K_m (K77) and K_m (Seidelmann) is at the 10^{-5} level, and its influence on $\Delta\psi_1$ and $\Delta\epsilon_1$ can be a few times 0.1 mas. If the computations are carried to 0.01 mas, some other terms are affected too. Due to its importance, the K value problem is discussed here in detail. K values are calculated from Eq. (2), in which the dynamical ellipticity of the Earth H is estimated by using the expression for lunisolar precession in longitude (see K77).

$$f_{T0} + P_g = 3H(K'_m\{(\mathscr{M}_0 - \mathscr{M}_2/2)\cos\epsilon_0 + \mathscr{M}_1\cos 2\epsilon_0/\sin\epsilon_0 + \mathscr{M}_3[\mathscr{M}_m/(\mathscr{M}_E + \mathscr{M}_m)](n_m^2/\omega\Omega)H(6\cos^2\epsilon_0 - 1)\} + K'_s S_0\cos\epsilon_0), \qquad (4)$$

where

$$K'_m = K_m/3H, \quad K'_s = K_s/3H.$$

The relative accuracy of H is proportional to that of f. In K77 the adopted value is $f_{1900} = 5037''.08 + 1''.10$, while in Seidelmann (1982) the f value is taken from Lieske *et al.* (1977). Both in calculating H and in deriving K from H one needs to use the sidereal mean motion of the Moon n_m and the Sun n_s and the mass ratio. These values in K77 are less accurate than in Seidelmann (1982). In Kinoshita and Souchay (1988) the K values are derived from the older H values given in K77 (which implies the use of K77's old n_m and n_s values, etc.) and new n_m and n_s values, etc. (from ELP2000 and VSOP82). This practice implies inconsistency in the evaluation of K.

By using ELP2000 and VSOP82 we recalculated H, K_m, and K_s values through two procedures. First, we simply use Eq. (4). The second procedure is more complicated: ϵ_0 in Eq. (4) is understood as the obliquity referred to the fixed ecliptic; an additional term E due to the reference plane difference is added in the Hamiltonian. These two procedures lead only to negligible differences. The newly estimated K_m, K_s values differ from those given by Seidelmann by only 5×10^{-7}; their effect on nutation is less than 0.01 mas. To avoid complications, it may be appropriate to simply adopt the K values of Seidelmann.

As pointed out by Kinoshita (1989), Eq. (4) did not include a second-order effect, which will affect the determination of K values. However, the second-order effect is at the level 10^{-6}, and its effect on nutation is also small (at most, some small effects arise in $\Delta\psi_1$ and $\Delta\epsilon_1$).

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V. SECOND-ORDER EFFECTS

As pointed out by Kubo (1982), second-order terms as large as 1.2 mas in $\Delta\psi_2$ and -0.2 mas in $\Delta\epsilon_2$ should be taken into account. Following Hori's (1966) formulas for perturbation theory, the second-order effect on nutation is

$$\delta_2\lambda = \{\lambda, W_2\} + 0.5\{\delta\lambda_1, W_1\}, \tag{5}$$

where $\{A, B\}$ is a Poisson bracket, and W_1 is a so-called determining function. In fact,

$$W_1 = \int U_{\text{period}}\, dt,$$

and

$$W_2 = \int \{U_{\text{secular}}, W_1\} dt + 0.5 \int \{U_{\text{period}}, W_1\}_{\text{period}}\, dt. \tag{6}$$

First, consider the $\{\delta\lambda_1, W_1\}/2$ term. Omitting here the detailed derivation, one obtains

$$\begin{aligned}\delta_2\psi = -0.5\Big\{&\Big[K\sum_v (i_5)\Big(\frac{E_v}{N_v}\Big)\cos\vartheta_v\Big]\\ &\times\Big[-K\sum_u\Big(\frac{E_u}{N_u}\Big)\sin\vartheta_u\Big]\\ &-\Big[K\sum_v\Big(\frac{\sin\vartheta_v}{N_v}\Big)\Big(\frac{\partial E_v}{\partial\epsilon}\Big)\Big]\\ &\times\Big[-\Big(\frac{K}{\sin\epsilon}\Big)\sum_u (i_5)\Big(\frac{B_u}{N_u}\Big)\cos\vartheta_u\Big]\Big\}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\delta_2\epsilon = -0.5\Big\{&\Big[K\sum_v (i_5)^2\Big(\frac{B_v}{\sin\epsilon}\Big)\frac{\sin\vartheta_v}{N_v}\Big]\\ &\times\Big[-K\sum_u\Big(\frac{E_u}{N_u}\Big)\sin\vartheta_u\Big]\\ &+\Big[K\sum_v (i_5)\Big\{\frac{\cos\vartheta_v}{N_v}\Big\}\partial\Big(\frac{B_v}{\sin\epsilon}\Big)\Big/\partial\epsilon\Big]\\ &\times\Big[-\Big(\frac{K}{\sin\epsilon}\Big)\sum_u (i_5)\Big(\frac{B_u}{N_u}\Big)\cos\vartheta_u\Big]\Big).\end{aligned} \tag{8}$$

For $(u,v) = (1,1)$, one has

$$\begin{aligned}\delta_2\psi &= +2.44 \text{ mas } \sin\Omega\cos\Omega =\\ &\quad +1.22 \text{ mas } \sin 2\Omega \Rightarrow \Delta\psi_2 \text{ term},\\ \delta_2\epsilon &= -0.24 \text{ mas } \cos 2\Omega \Rightarrow \Delta\epsilon_2 \text{ term}.\end{aligned} \tag{9}$$

These results are similar to those in Kubo (1982), Kubo and Fukushima (1988), and Kinoshita (1988). Besides $(u,v) = (1,1)$, other terms like $(u,v) = (1,9)$ should also be considered, which lead to

$$\delta_2\psi = 0.137 \text{ mas } \sin(\vartheta_9 + \Omega) - 0.071 \text{ mas } \sin(\vartheta_9 - \Omega) \tag{10}$$

and

$$\delta_2\epsilon = 0.049\cos(\vartheta_9 - \Omega) - 0.025\cos(\vartheta_9 + \Omega).$$

These terms are not included in the 106 terms of the IAU (1980) nutation series. There arguments and periods are

$(\vartheta_9 + \Omega)$, $v(0,0,2,-2,3)$, period 187.66 days,

$(\vartheta_9 - \Omega)$, $v(0,0,2,-2,1)$, period 177.84 days.

Similarly, $(u,v) = (1,2)$ and $(1,31)$ will produce an effect on the order of 0.01 mas, which should also be included if 0.01 mas accuracy is required. Now, we turn to W_2: as proved by Kubo (1982, Appendix), $\{U_{\text{secular}}, W_1\}dt$ should be neglected. So, we consider $\{U_{\text{period}}, W_1\}_{\text{period}}\, dt$. Omitting again the detailed derivation, the result is

$$\begin{aligned}\{U_{\text{period}}, W_1\} = -\Big\{&\Big[K^*\sum_v (i_5)B_v\sin\vartheta_v\Big]\\ &\times\Big[K\sum_u\Big(\frac{E_u}{N_u}\Big)\sin\vartheta_u\Big]\\ &+\sum_v\Big[K(i_5)\Big(\frac{B_V}{N_v}\Big)\cos\vartheta_v\Big]\\ &\times\Big[K^*\sum_u E_u\cos\vartheta_u\Big]\Big\},\end{aligned} \tag{11}$$

where $K^* = KG$, and G is the total angular momentum of the Earth. For $(u,v) = (1,1)$, the right-hand side of Eq. (11) is $-KK^*(B_1E_1/N_1)$, which is secular in nature, hence $\{\ \}_{\text{period}} = 0$. This has already been pointed out by Kubo (1982). We are only interested in $(u,v) = (1,9)$, $(1,2)$, and $(1,31)$; for $(9,1)$, Eq. (11) leads to

$$\begin{aligned}\delta_2\psi &= -0.022\sin(\vartheta_9 - \Omega) - 0.020\sin(\vartheta_9 + \Omega),\\ \delta_2\epsilon &= \ \ 0.024\cos(\vartheta_9 - \Omega) + 0.08\cos(\vartheta_9 + \Omega).\end{aligned}$$

Combined with the above results, the final second-order effects are

$$\begin{aligned}\delta_2\psi = &\ 1.22\sin 2\Omega - 0.09\sin(\vartheta_9 - \Omega)\\ &+ 0.12\sin(\vartheta_9 + \Omega)\\ &- 0.02\sin 3\Omega + 0.02\sin(\vartheta_{31} + \Omega)\\ &- 0.01\sin(\vartheta_{31} - \Omega),\\ \delta_2\epsilon = &-0.24\cos 2\Omega + 0.07\cos(\vartheta_9 - \Omega)\\ &- 0.02\cos(\vartheta_9 + \Omega).\end{aligned} \tag{12}$$

Here the unit is mas. The last three terms in $\delta_2\psi$ are due to $(v,u) = (1,2)$ and $(1,31)$.

In the above derivation, only second-order Poisson brackets of Andoyer variables have been taken into account. The contribution of Delaunay variables should also be added (Kinoshita 1989). In our computation, the numerical values of its effect on $\Delta\psi_1$ and $\Delta\epsilon_1$ were taken from Kinoshita (1988). Similar results can be derived from Aoki's Eqs. (D.6) and (D.7), where $\partial h^*/\partial I$ term should be removed since it has already been included in the first-order solution. Terms like $\dot{h}^*(\partial\Delta h/\partial h)$ should also be excluded, as they belong to $\{U_{\text{secular}}, W_1\}dt$.

VI. "GEOPHYSICAL" APPROACH FOR NUTATION DETERMINATION

As emphasized by Melchior (1971), lunisolar torque, the driving force of nutation, is directly related to the tesseral part of the tidal potential. Thus there is a possibility to derive nutation from known tesseral (diurnal) tidal waves. The equations given in (Melchior 1983) are

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$$\Delta\epsilon_v = -E_m \sum_v \left(\frac{\omega}{\Delta\omega_v}\right)(P_v - P_{-v})\cos(\Delta\omega_v t),$$

$$\Delta\psi_v = -\left(\frac{E_m}{\sin\epsilon}\right)\sum_v \left(\frac{\omega}{\Delta\omega_v}\right)(P_v + P_{-v})\sin(\Delta\omega_v t). \quad (13)$$

The relation between E_m and K_m is

$$E_m = K_m/2\omega = 0''.016\,443\,105,$$

where ω is the frequency of one sidereal day (units of radians/Julian century). $\Delta\omega_v$ is the frequency of nutation, which is directly related to N_v in Eq. (1).

Two tidal waves with $\omega_v = \omega + \Delta\omega_v$ and $\omega_{-v} = \omega - \Delta\omega_v$ (symmetric with respect to the K_1 wave) with amplitude P_v and P_{-v} compose one nutation term with frequency $\Delta\omega_v$.

Both A_v in Eq. (1) and P_v are derived from orbital elements of the Sun and Moon. The relations between A_v and P_v can be derived. They depend on the (i_5) value in the nutation argument. Omitting once more the detailed derivations, the final results are (P_v has the same unit as in Cartwright and Taylor 1971):

For $(i_5) = 0$,

$$P_v = P_{-v} = -A_v^0(\sin 2\epsilon)/2, \quad (14a)$$
$$A_v^0 = -(P_v + P_{-v})/\sin 2\epsilon.$$

For $(i_5) = 1$,

$$P_v = -A_v^1(\cos 2\epsilon - \cos\epsilon),$$
$$P_{-v} = -A_v^1(\cos 2\epsilon + \cos\epsilon),$$
$$(P_v)/P_{-v} = (\cos 2\epsilon - \cos\epsilon)/(\cos 2\epsilon + \cos\epsilon), \quad (14b)$$
$$A_v^1 = -(P_v + P_{-v})/2\cos 2\epsilon.$$

For $(i_5) = 2$,

$$P_v = -(A_v^2/2)\sin\epsilon(1 - \cos\epsilon),$$
$$P_{-v} = (A_v^2/2)\sin\epsilon(1 + \cos\epsilon),$$
$$(P_v/P_{-v}) = -(1 - \cos\epsilon)/(1 + \cos\epsilon), \quad (14c)$$
$$A_v^2 = 2(P_v + P_{-v})/\sin 2\epsilon.$$

If the arguments for diurnal tidal wave ω_v are

$$1(t),\ j_1(s),\ j_2(h),\ j_3(p),\ j_4(N'),\ j_5(p_s),$$

the related nutation $\Delta\omega_v$ has the following arguments:

$$i_1(1),\ i_2(1'),\ i_3(F),\ i_4(D),\ i_5(\Omega),$$

then the relations between j and i are, from the tidal to the nutation argument,

$$i_1 = -j_3,\quad i_2 = -j_5,\quad i_3 = (j_1 - 1) + j_2 + j_3 + j_5,$$
$$i_4 = -j_2 - j_5,\quad i_5 = (j_1 - 1) + j_2 + j_3 - j_4 + j_5;$$

and from nutation to tidal argument,

$$j_1 = i_1 + i_3 + i_4 + 1,\quad j_2 = i_2 - i_4,\quad j_3 = -i_1,$$
$$j_4 = +i_3 - i_5,\quad j_5 = -i_2.$$

The above Eqs. (14a)–(14c) are approximate in the following sense:

(1) The equations are correct only when

$A_v^i = 0$ for $(i_5) \neq i$, e.g., $A_v^0 = 0$ for $(i_5) \neq 0$, etc.
This condition is fulfilled when the effect of secular perturbations and planetary perturbation on nutation can be ignored. This is true in most cases at the 10^{-7} level. The only exceptions are $v = 1$, 31, and 32, and even then the maximum relative error is less than 5×10^{-5}.

(2) The above results are based on a second-order tidal potential only. For the third-order tides, a coefficient ratio J_3/J_2 must be considered.

(3) All the aforementioned values for $\Delta\epsilon$, $\Delta\psi$ are those related to the angular-momentum axis.

VII. OPPOLZER TERM—RESPONSE OF CHANDLER WOBBLE MODE

The response of the Earth to the tidal potential can be expanded as a sum of the Earth's normal mode eigenfunctions (Wahr 1981b). For a rigid Earth, only two modes are relevant: tiltover mode (TOM) and Chandler Wobble mode (CW); the latter is actually the Eulerian free nutation in the rigid case. If only the amplitude of nutation ξ_v is of interest, it can be written symbolically

$$\xi_v = \zeta_v[1/(\omega - \omega_v) - 1/(\omega_c - \omega_v)], \quad (15)$$

where $\omega_c = -\omega(C - A)/A$ is the Eulerian frequency. The two terms in parentheses are the response of TOM and CW, respectively. It is easy to prove that $a_v = \zeta_v/(\omega - \omega_v)$ is the nutation angular-momentum axis, and by including the $-a_v[(\omega - \omega_v)/(\omega_c - \omega_v)]$ term, ξ_v becomes the nutation of the figure axis.

As mentioned in Sec. VI, the two tidal waves with $\omega_{+v} = \omega + \Delta\omega_v$, $\omega_{-v} = \omega - \Delta\omega_v$ together form a nutation with frequency $\Delta\omega_v$. Consequently,

$$\sin\epsilon\Delta\psi_v = -(a_v - a_{-v}),$$
$$\Delta\epsilon_v = -(a_v + a_{-v}), \quad (16)$$

here $\Delta\psi_v$, $\Delta\epsilon_v$ are understood only as amplitudes. From Eq. (15), the (CW) contributions to $\Delta\psi_v \sin\epsilon$ and $\Delta\epsilon_v$ are

$$[\Delta\omega_v/(\omega_c'^2 - \Delta\omega_v^2)]\{\Delta\epsilon_v\omega_c' + \Delta\psi\sin\epsilon\Delta\omega_v\}:$$
(CW) in $\Delta\psi_v \sin\epsilon$,
$$[\Delta\omega_v/(\omega_c'^2 - \Delta\omega_v^2)]\{\Delta\epsilon_v\Delta\omega_v + \Delta\psi\sin\epsilon\omega_c'\}:$$
(CW) in $\Delta\epsilon_v$. (17)

After substituting Eq. (1), $\Delta\psi_v = KE_v/N_v$, and $\Delta\epsilon_v = -(K/\sin\epsilon)(i_5)B_v/N_v$ into Eq. (17), and noting that $(i_5) = i$ for A_v^i, one readily sees that the (CW) contributions of Eq. (17) are exactly the Oppolzer terms in K77. The original equation in K77 for the Oppolzer terms are only suitable with a second-order tidal potential, while Eq. (17) here is also correct for a third-order one.

VIII. EXPANSION OF NUTATION SERIES

By using the results of Secs. VI and VII, the nutation series can be set up from given tidal-potential waves, e.g., those computed by Cartwright and Taylor (1971) or the ones given by Xi and Hou (1987). For the original 106 terms, the agreement between this "geophysical" approach and that of K77 is fairly good. Except for a few periods, the differences are only at 0.01 mas. The geophysical procedure is good enough to derive those small-amplitude nutation terms at the 0.01 mas accuracy level.

As we mentioned earlier, 106 terms are chosen in the IAU 1980 nutation theory. The criterion of the choice is a level of 0.05 mas in nutation amplitude. Since our computation is carried to 0.01 mas, more terms should be added. Terms due to the second-order tidal potential could be computed both

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TABLE I. Nutation of the rigid Earth with original 106 terms (J2000).

	ARGUMENTS	PERIODS	PSI	EPSILON	RATE(PSI)	RATE(EPSILON)
	l l'F D Ω	(days)	(0.0001 a.s.)		(0.0001 a.s.per cy)	
1	0 0 0 0 1	-6798.38	-172819.5	92285.6	-174.4	9.2
2	0 0 0 0 2	-3399.19	2090.9	-903.7	0.2	0.5
3	-2 0 2 0 1	1305.48	45.1	-24.1	0.1	0.0
4	2 0-2 0 0	-1095.18	11.1	0.0	0.0	0.0
5	-2 0 2 0 2	1615.75	-3.1	1.4	0.0	0.0
6	1-1 0-1 0	-3232.86	-3.3	0.0	0.0	0.0
7	0-2 2-2 1	6786.32	-1.5	0.8	0.0	0.0
8	2 0-2 0 1	-943.23	0.7	-0.4	0.0	0.0
9	0 0 2-2 2	182.62	-12775.6	5533.7	-1.3	-2.9
10	0 1 0 0 0	365.26	1255.2	-1.4	-3.0	0.0
11	0 1 2-2 2	121.75	-499.4	216.2	1.2	-0.7
12	0-1 2-2 2	365.22	213.2	-92.4	-0.5	0.3
13	0 0 2-2 1	177.84	126.0	-67.1	0.1	-0.0
14	2 0 0-2 0	205.89	44.9	-0.1	0.0	0.0
15	0 0 2-2 0	173.31	-20.6	0.1	0.0	0.0
16	0 2 0 0 0	182.63	15.8	-0.0	-0.1	0.0
17	0 1 0 0 1	386.00	-14.8	7.9	0.0	-0.0
18	0 2 2-2 2	91.31	-15.2	6.6	0.1	-0.0
19	0-1 0 0 1	-346.64	-9.8	5.2	0.0	-0.0
20	-2 0 0 2 1	-199.84	-5.2	2.8	-0.0	0.0
21	0-1 2-2 1	346.60	-4.9	2.6	0.0	0.0
22	2 0 0-2 1	212.32	4.0	-2.1	0.0	0.0
23	0 1 2-2 1	119.61	3.5	-1.9	0.0	0.0
24	1 0 0-1 0	411.78	-2.8	0.0	0.0	0.0
25	2 1 0-2 0	131.67	1.0	0.0	0.0	0.0
26	0 0-2 2 1	-169.00	0.8	-0.4	0.0	0.0
27	0 1-2 2 0	-329.79	-0.8	0.0	0.0	0.0
28	0 1 0 0 2	409.23	0.7	-0.3	0.0	0.0
29	-1 0 0 1 1	-388.27	0.6	-0.3	0.0	0.0
30	0 1 2-2 0	117.54	-0.6	0.0	0.0	0.0
31	0 0 2 0 2	13.66	-2215.3	949.2	-0.2	-0.5
32	1 0 0 0 0	27.55	677.7	-9.7	0.1	0.0
33	0 0 2 0 1	13.63	-378.5	194.1	-0.4	0.0
34	1 0 2 0 2	9.13	-295.9	126.1	-0.0	-0.1
35	1 0 0-2 0	-31.81	-149.6	-1.9	-0.0	0.0
36	-1 0 2 0 2	27.09	119.2	-51.4	0.0	0.0
37	0 0 0 2 0	14.77	60.5	-1.6	0.0	0.0
38	1 0 0 0 1	27.67	61.1	-31.9	0.1	0.0
39	-1 0 0 0 1	-27.44	-54.3	29.7	-0.1	0.0
40	-1 0 2 2 2	9.56	-58.5	24.9	-0.0	-0.0
41	1 0 2 0 1	9.12	-50.9	25.7	-0.1	0.0
42	0 0 2 2 2	7.10	-38.1	16.2	0.0	-0.0
43	2 0 0 0 0	13.78	27.9	-0.8	0.0	0.0
44	1 0 2-2 2	23.94	27.6	-11.9	0.0	0.0
45	2 0 2 0 2	6.86	-30.7	13.0	0.0	-0.0
46	0 0 2 0 0	13.61	24.7	-0.7	0.0	0.0
47	-1 0 2 0 1	26.98	19.8	-10.3	0.0	0.0
48	-1 0 0 2 1	31.96	14.7	-7.7	0.0	0.0
49	1 0 0-2 1	-31.66	-12.1	6.6	-0.0	0.0
50	-1 0 2 2 1	9.54	-10.1	5.1	-0.0	0.0
51	1 1 0-2 0	-34.85	-7.0	-0.1	0.0	0.0
52	0 1 2 0 2	13.17	7.4	-3.2	-0.0	0.0
53	0-1 2 0 2	14.19	-6.9	3.0	0.0	-0.0

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TABLE I. (continued)

	ARGUMENTS	PERIODS	PSI	EPSILON	RATE(PSI)	RATE(EPSILON)
	l l'F D Ω	(days)	(0.0001 a.s.)		(0.0001 a.s.per cy)	
54	1 0 2 2 2	5.64	-7.7	3.2	0.0	0.0
55	1 0 0 2 0	9.61	6.3	-0.3	0.0	0.0
56	2 0 2-2 2	12.81	6.3	-2.7	0.0	0.0
57	0 0 0 2 1	14.80	-6.2	3.2	-0.0	0.0
58	0 0 2 2 1	7.09	-6.6	3.3	-0.0	0.0
59	1 0 2-2 1	23.86	5.6	-2.9	0.0	0.0
60	0 0 0-2 1	-14.73	-4.6	2.6	0.0	0.0
61	1-1 0 0 0	29.80	4.5	-0.1	-0.0	0.0
62	2 0 2 0 1	6.85	-5.3	2.6	-0.0	0.0
63	0 1 0-2 0	-15.39	-4.2	-0.1	0.0	0.0
64	1 0-2 0 0	-26.88	3.9	0.1	0.0	0.0
65	0 0 0 1 0	29.53	-3.8	0.1	0.0	0.0
66	1 1 0 0 0	25.62	-3.2	0.1	0.0	0.0
67	1 0 2 0 0	9.11	3.2	-0.1	0.0	0.0
68	1-1 2 0 2	9.37	-2.8	1.2	0.0	0.0
69	-1-1 2 2 2	9.81	-2.8	1.2	0.0	0.0
70	-2 0 0 0 1	-13.75	-2.1	1.2	0.0	0.0
71	3 0 2 0 2	5.49	-2.9	1.2	0.0	0.0
72	0-1 2 2 2	7.24	-2.6	1.1	0.0	0.0
73	1 1 2 0 2	8.91	2.4	-1.0	-0.0	0.0
74	-1 0 2-2 1	-32.61	-1.9	1.0	0.0	0.0
75	2 0 0 0 1	13.81	2.1	-1.1	0.0	0.0
76	1 0 0 0 2	27.78	-1.9	0.8	0.0	0.0
77	3 0 0 0 0	9.18	1.5	-0.1	0.0	0.0
78	0 0 2 1 2	9.34	1.6	-0.7	0.0	0.0
79	-1 0 0 0 2	-27.33	1.3	-0.6	0.0	0.0
80	1 0 0-4 0	-10.08	-1.3	-0.1	0.0	0.0
81	-2 0 2 2 2	14.63	1.3	-0.6	0.0	0.0
82	-1 0 2 4 2	5.80	-1.5	0.6	0.0	0.0
83	2 0 0-4 0	-15.91	-1.2	-0.0	0.0	0.0
84	1 1 2-2 2	22.47	1.3	-0.5	0.0	0.0
85	1 0 2 2 1	5.64	-1.3	0.7	0.0	0.0
86	-2 0 2 4 2	7.35	-1.2	0.5	0.0	0.0
87	-1 0 4 0 2	9.06	1.1	-0.5	0.0	0.0
88	1-1 0-2 0	-29.26	0.9	0.0	0.0	0.0
89	2 0 2-2 1	12.79	1.0	-0.5	0.0	0.0
90	2 0 2 2 2	4.68	-1.1	0.5	0.0	0.0
91	1 0 0 2 1	9.63	-1.0	0.5	0.0	0.0
92	0 0 4-2 2	12.66	0.9	-0.4	0.0	0.0
93	3 0 2-2 2	8.75	0.9	-0.4	0.0	0.0
94	1 0 2-2 0	23.77	-0.7	0.0	0.0	0.0
95	0 1 2 0 1	13.14	0.8	-0.4	0.0	0.0
96	-1-1 0 2 1	35.03	0.7	-0.4	0.0	0.0
97	0 0-2 0 1	-13.58	-0.6	0.3	0.0	0.0
98	0 0 2-1 2	25.42	-0.7	0.3	0.0	0.0
99	0 1 0 2 0	14.19	-0.6	0.0	0.0	0.0
100	1 0-2-2 0	-9.53	-0.6	-0.0	0.0	0.0
101	0-1 2 0 1	14.16	-0.7	0.3	0.0	0.0
102	1 1 0-2 1	-34.67	-0.6	0.3	0.0	0.0
103	1 0-2 2 0	32.76	-0.6	0.0	0.0	0.0
104	2 0 0 2 0	7.13	0.6	-0.0	0.0	0.0
105	0 0 2 4 2	4.79	-0.7	0.3	0.0	0.0
106	0 1 0 1 0	27.32	0.5	-0.0	0.0	0.0

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TABLE II. Nutation of the rigid Earth, additional terms (J2000).

	ARGUMENTS	PERIODS	PSI	EPSILON		ARGUMENTS	PERIODS	PSI	EPSILON
	l l'F D Ω	(days)	(0.00001 a.s.)			l l'F D Ω	(days)	(0.00001 a.s.)	
1	3 0 2 2 2	4.003	-1	1	79	0-2 0 0 1	-177.852	-1	1
2	1 0 2 4 2	4.083	-2	1	80	0 0 2-2 3	187.662	12	-2
3	4 0 2 0 2	4.579	-3	1	81	2 0 0-2 2	219.167	-3	1
4	2 0 2 2 1	4.680	-2	1	82	-2 1 2 0 1	285.406	-1	0
5	0 0 2 4 1	4.789	-1	1	83	-2 1 2 0 2	297.913	-1	0
6	3 0 2 0 1	5.488	-5	2	84	-1 0 2-1 1	313.042	-4	1
7	1 1 2 2 2	5.557	1	-1	85	0-1 0 0 2	-329.819	3	-1
8	1-1 2 2 2	5.731	-6	2	86	1 0 0-1 1	438.335	2	-1
9	-1 0 2 4 1	5.797	-3	1	87	-2-1 2 0 2	-471.950	4	-2
10	-1-1 2 4 2	5.895	-2	1	88	-2-1 2 0 1	-507.157	4	0
11	2 1 2 0 2	6.733	4	-2	89	-3 0 2 1 2	-552.625	2	-1
12	0 0 4 0 2	6.817	2	-1	90	0 0 0 0 3	-2266.120	-2	0
13	2 0 2 0 0	6.846	3	-0	91	-1-1 2-1 2	3230.131	13	-5
14	0 1 2 2 2	6.961	5	-2	92	-1 0 1 0 1	3231.495	-15	3
15	1 0 2 1 2	6.976	3	-1	93	-1 0 1 0 2	6159.135	3	-1
16	2-1 2 0 2	6.991	-5	2	94	-1 1 0 1 1	6164.100	7	-4
17	0 0 2 2 0	7.081	4	-0	95	2-1 2 2 2	4.744	-1	0
18	2 0 0 2 1	7.135	-1	1	96	0-1 2 4 2	4.856	-1	0
19	0-1 2 2 1	7.229	-4	2	97	3-1 2 0 2	5.576	-1	0
20	-2 0 2 4 1	7.341	-2	1	98	1-1 2 2 1	5.726	-1	0
21	0-2 2 2 2	7.383	-1	1	99	1 0 0 4 0	5.823	1	0
22	0 0 0 4 0	7.383	5	-0	100	4 0 2-2 2	6.638	1	0
23	0 0 0 4 1	7.391	-2	1	101	2 1 2 0 1	6.726	1	0
24	1 0 4-2 2	8.676	2	-1	102	4 0 0 0 0	6.889	1	0
25	3 0 2-2 1	8.734	2	-1	103	0 1 2 2 1	6.953	1	0
26	1 1 2 0 1	8.898	4	-2	104	1 0 2 1 1	6.968	1	0
27	-1 0 4 0 1	9.045	2	-1	105	2-1 2 0 1	6.983	-1	0
28	0 1 2 1 2	9.107	-2	1	106	-1 0 2 3 2	7.220	1	0
29	-3 0 0 0 1	-9.172	-1	1	107	-2-1 2 4 2	7.500	-1	0
30	-1 1 2 2 2	9.313	6	-2	108	0-1 0 4 0	7.535	1	0
31	0 0 2 1 1	9.327	3	-1	109	2 0 2-1 2	8.935	-1	0
32	1-1 2 0 1	9.354	-4	2	110	1 0 2 0-1	9.096	1	0
33	-1 0 0-2 1	-9.600	-4	3	111	3 0 0 0 1	9.197	1	0
34	-1-1 2 2 1	9.799	-5	2	112	-1 1 2 2 1	9.300	1	0
35	1-1 0 2 0	9.874	5	-0	113	1 1 0 2 0	9 367	-1	0
36	1 0 0-4 1	-10.070	-1	1	114	1-1 0 2 1	9.888	-1	0
37	-1 0 0 4 1	10.100	-2	1	115	-1-2 2 2 2	10.085	-1	0
38	-1-1 0 4 0	10.371	1	-0	116	2 1 2-2 1	12.354	1	0
39	2 1 2-2 2	12.377	3	-1	117	0 2 2 0 2	12.710	1	0
40	0 0 4-2 1	12.639	2	-1	118	2 0 2-2 0	12.763	-1	0
41	1 0 2-1 2	13.222	-3	1	119	-2 0 4 0 2	13.492	-1	0
42	2 1 0 0 0	13.276	-3	0	120	1 1 0 1 0	13.719	1	0
43	0 0 2 0 1	13.634	-1	0	121	2 0 0 0 2	13.833	-1	0
44	0 0 2 0 3	13.688	2	0	122	-1 0 2 1 2	14.130	1	0
45	0 1 0 2 1	14.221	2	-1	123	0 1 0 2-1	14.162	-1	0
46	1 0 0 1 0	14.254	-3	0	124	0-2 2 0 2	14.765	-1	0
47	2-1 0 0 0	14.317	4	-0	125	2 0-2 2-1	14.934	1	0
48	-2 0 2 2 1	14.600	2	-1	126	-2-1 2 2 2	15.242	1	0
49	0 0 0-2 2	-14.701	1	-1	127	-1 0 0 3 0	15.314	-1	0
50	0 0 0 2 2	14.830	-5	2	128	-2-1 0 4 0	16.630	1	0
51	0 1 0-2 1	-15.353	-3	2	129	-1 0 4-2 2	23.430	-1	0
52	0-1 0 2 1	15.422	-2	1	130	1 2 0 0 0	23.942	-1	0
53	2 0 0-4 1	-15.869	-1	1	131	-1 1 2 0 1	25.129	1	0
54	-2 0 0 4 1	15.943	1	-1	132	0 0 2-1 1	25.325	-1	0
55	0-2 0 2 0	16.064	2	-0	133	1-1 2-2 1	25.525	-1	0
56	0 0 2-4 1	-16.102	-1	1	134	1-1 2-2 2	25.622	-1	0
57	1 1 2-2 1	22.395	3	-1	135	2 0 0-1 0	25.826	1	0
58	-1 1 2 0 2	25.222	4	-2	136	-1 2 0 2 0	27.093	-1	0
59	-1-1 0 0 1	-25.525	2	-1	137	0-1 2-1 2	27.321	1	0
60	1 1 0 0 1	25.719	-3	2	138	3 0-2 0-1	28.149	1	0
61	1 0-2 0 1	-26.772	3	-1	139	-1-1 2 0 1	29.138	-1	0
62	0 0 1 0 1	27.322	-2	0	140	-1 1 0 2-1	29.138	-1	0
63	-1-1 2 0 2	29.263	-2	1	141	-3 0 2 2 1	31.055	1	0
64	-1 1 0 2 1	29.390	-1	1	142	1-2 0 0 0	32.451	1	0
65	0 0 0-1 1	-29.403	3	-2	143	-2 0 0 3 0	34.475	-1	0
66	0 0 0 1 1	29.659	-4	2	144	-3 0 0 4 0	37.625	1	0
67	-1 1 0 0 1	-29.673	-2	2	145	-1 0-2 4-2	38.522	-1	0
68	1-1 0 0 1	29.934	5	-3	146	-1 0-2 4 0	38.964	-1	0
69	1 0 0-2 2	-31.517	3	-1	147	-1-1-2 4-2	43.064	1	0
70	-1 0 0 2 2	32.112	-4	2	148	1 1 2-4 1	-43.339	-1	0
71	-1 0 2-2 2	-32.451	3	-1	149	0 2 2-2 1	90.103	1	0
72	-1 1 2-2 1	-35.803	-1	1	150	2 0 2-4 2	96.780	-1	0
73	-1-2 0 2 0	38.522	3	-0	151	2 1 0-2 1	134.272	1	0
74	1 0 2-4 1	-38.742	-4	2	152	-2 0 4-2 2	156.521	-1	0
75	0 3 2-2 2	73.051	-5	2	153	-2 2 2 0 2	164.083	-1	0
76	0 3 0 0 0	121.753	3	-0	154	0 2 0 0 1	187.671	-1	0
77	-2-1 0 2 1	-129.169	-2	1	155	1-1 2-3 2	193.555	1	0
78	0 0 2-2 1	177.844	-9	7	156	-4 0 2 2 2	-235.960	-1	0

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by K77's procedure or by the geophysical one. The results should be equivalent to 0.01 mas. To avoid time-consuming derivations for expanding K77's procedure to the third-order tidal potential, these terms can be determined simply by the geophysical procedure, in which only the coefficient of the J_3/J_2 ratio should be accounted for. Concerning third-order diurnal tidal waves, although the largest amplitude P in the potential is the wave 155.555, its effect on nutation is only 0.02 mas, since $\omega/\Delta\omega$ is comparatively small. On the contrary,the amplitude of wave 165.455 is smaller, but $\omega/\Delta\omega$ is larger; it produces the largest third-order nutation amplitude at a period of 3231.5 days. Besides the original 106 terms, there are altogether more than 150 terms with nutation amplitudes $\geqslant$0.005 mas.

IX. SUGGESTION OF IMPROVED RIGID-EARTH NUTATION EVALUATION

Combining all the aforementioned corrections and improvements, a new series of nutation of the rigid earth has been compiled for the figure axis. It comprises two tables. Table I contains the original 106 period terms. In this table the difference between Brown's table, as improved by Eckert *et al.* (1966), and ELP 2000 has been taken into account. The K values are practically the same as those of Seidelmann. Second-order perturbations with frequencies within the 106 terms are included. The results are rounded to 0.01 mas. In addition, the rate dN_v/dt has been accounted for, which affects basically only the rate of $\Delta\psi_1$ and $\Delta\epsilon_1$. Table II lists all terms other than the original 106 periods, in which 16 terms have amplitudes $\geqslant$0.05 mas.

Besides the values in these two tables, the nutation due to the effect of planetary pertubation should also be taken into account. Those terms were first derived by Vondrack (1982a,b,c) and later improved by Kinoshita and Souchay (1988).

As far as the accuracy of our series is concerned, for most individual terms the standard errors are 0.01 mas. Short-period terms are more accurate than long-period ones, since the same error in A_v and P_v will be magnified by different scales $\omega/\Delta\omega_v$. Accordingly, the terms like $P = 3230$ day may have an error up to 0.05 mas. The accuracy of $\Delta\psi_1$ and $\Delta\epsilon_1$ is much worse, since it depends strongly on the accuracy of K_m, which is directly influenced by the error in the adopted precession value. Recent VLBI observations indicated that the currently adopted precession might have an error of $-0''.2$/century, which implies possible errors of 0.7 mas in $\Delta\psi_1$ and -0.4 mas in $\Delta\epsilon_1$.

Compared with VLBI observed nutation corrections one can see that our new rigid nutation results could only remove about 10% of the discrepancy in the 18.6 yr term and semi-annual term (due to the contribution of two nearly semi-annual terms). The remaining discrepancy can only be explained geophysically. In view of recent geophysical studies, it seems necessary to revise Wahr's theory (1981a). For instance, the FCN frequency should be revised, and the effect of ocean tides be added. It is planned to derive a revised Wahr equation in order to apply our improved rigid nutation series to the deformable Earth. Then the new nutation series for the deformable Earth will be compared with the IAU series.

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